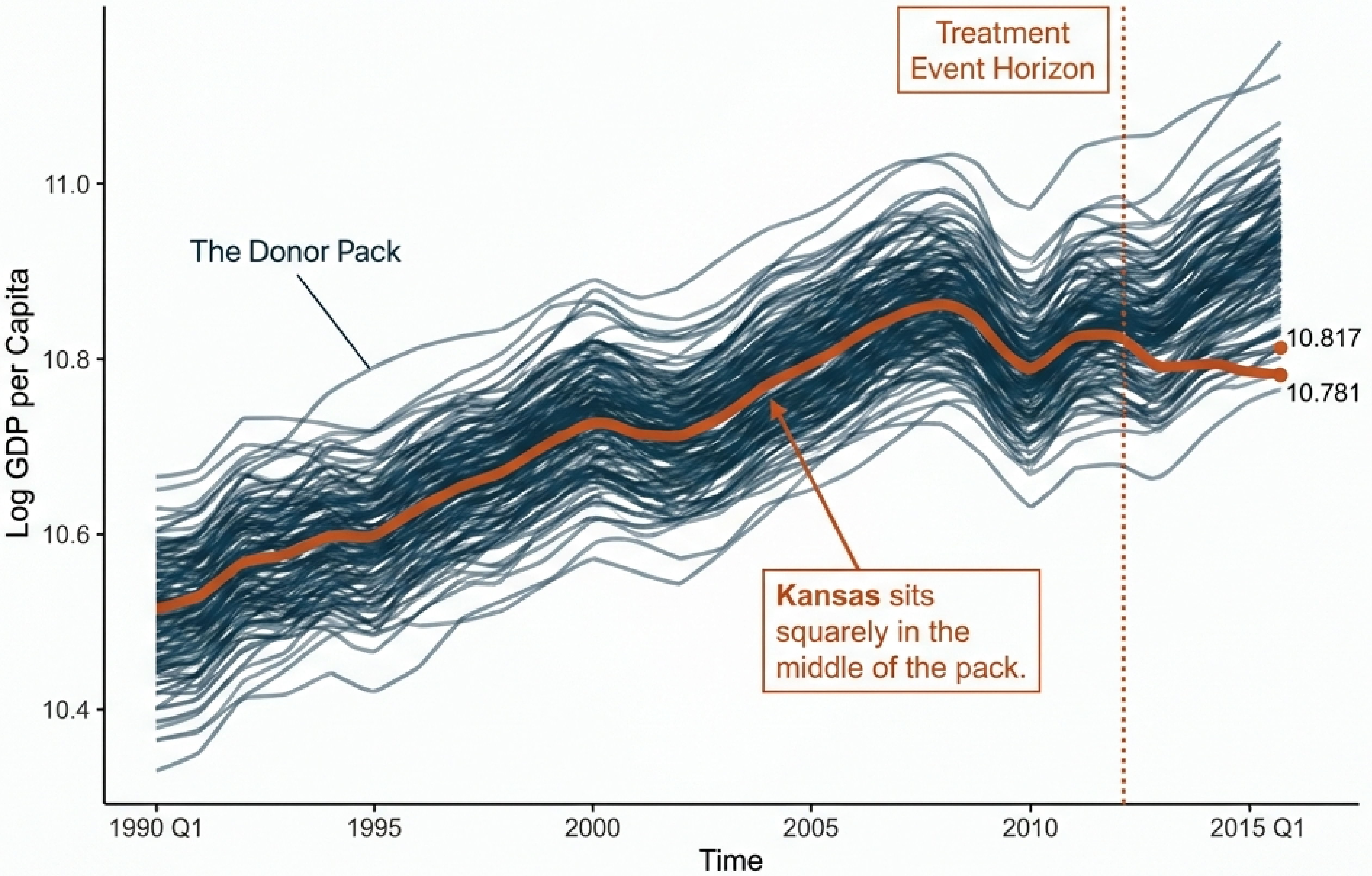
The Augmented Synthetic Control Method

A Methodological Guide: De-biasing Counterfactuals with the 2012 Kansas Tax Cuts

Focus	Estimator	Application
Methodological framework, implementation steps, and inferential validation.	`augsynth()` via Ridge and Covariate Augmentation.	Single treated unit ($N_{tr} = 1$).

The Missing Counterfactual: Kansas, 2012



The Problem

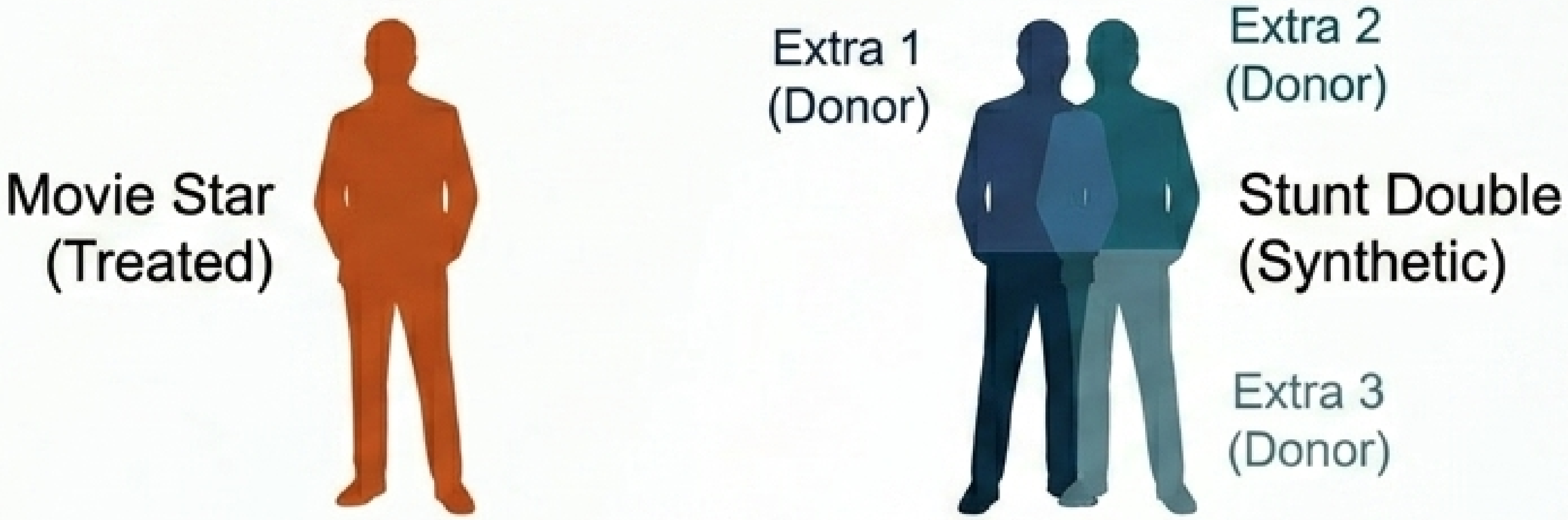
There is no single state whose trajectory perfectly mirrors Kansas pre-2012.

To isolate the effect of the tax cut on log GDP per capita, we cannot pick a "most similar" state.

We must construct a synthetic baseline.

Classic SCM and the ATT Estimand

Analogy: The Stunt Double



The synthetic unit mimics the treated unit safely before the dangerous scene (treatment).

The distance between them during the scene is the causal effect.

The ATT ($\hat{\tau}_t$)

$$\hat{\tau}_t = Y_{1t} - \sum\left(\hat{\gamma}_i^{\text{scm}} * Y_{it}\right), \text{ for } t > T_0$$

1 Average Treatment effect on the Treated (ATT) at time t

2 Actual Kansas GDP

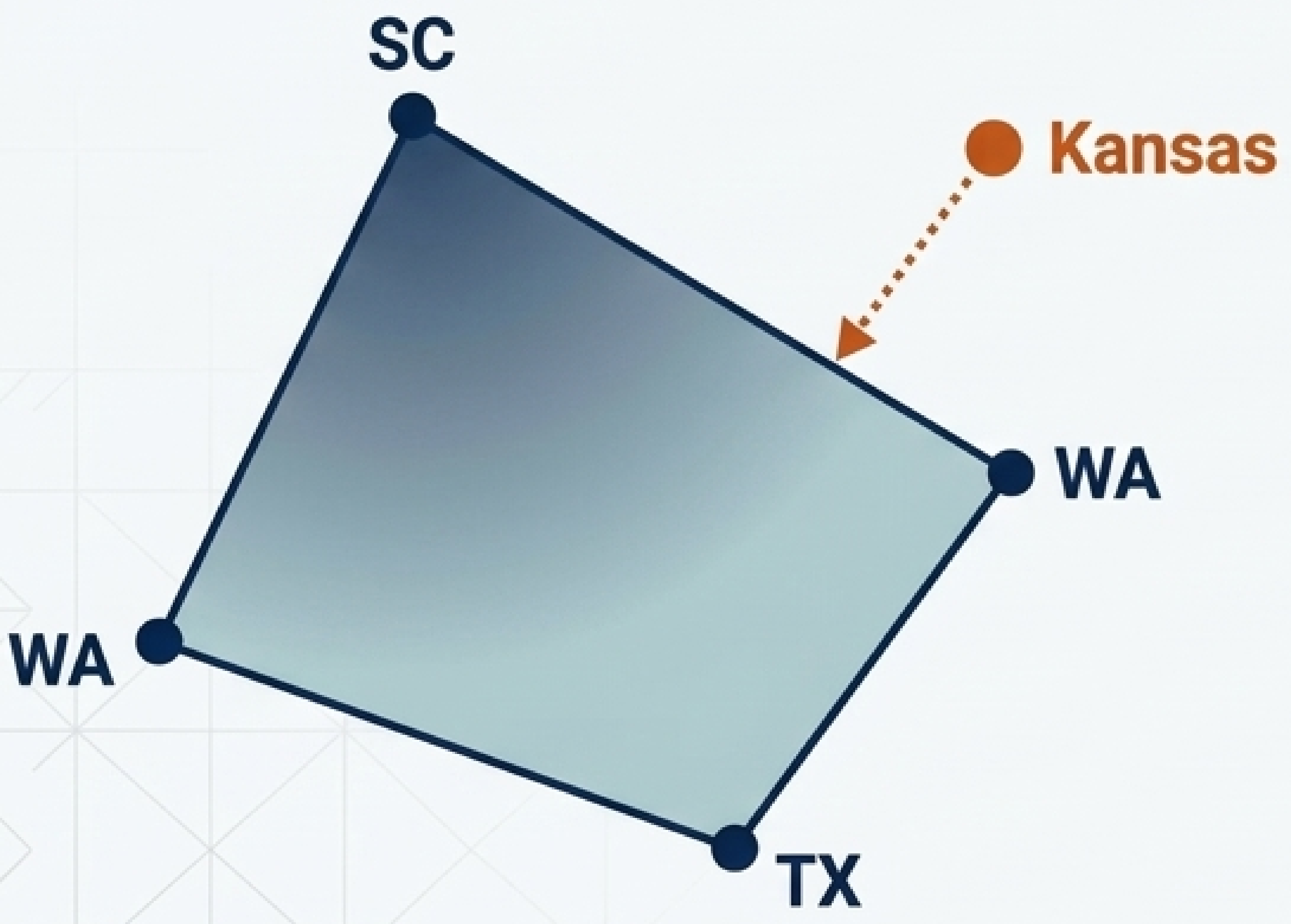
3 Synthetic Kansas / The weighted sum of donor outcomes

NotebookLM

The Simplex Constraint and the Convex Hull

Analogy: Mixing Paint

You can blend stocked colors to get any shade between them (interpolation). You cannot use a negative amount of blue to get a brighter blue (extrapolation).



Mathematical Constraints

SCM Optimization

$$\arg\min_{\gamma} \| X_1 - X_0{}' \gamma \|_2^2$$

The Rigid Rules

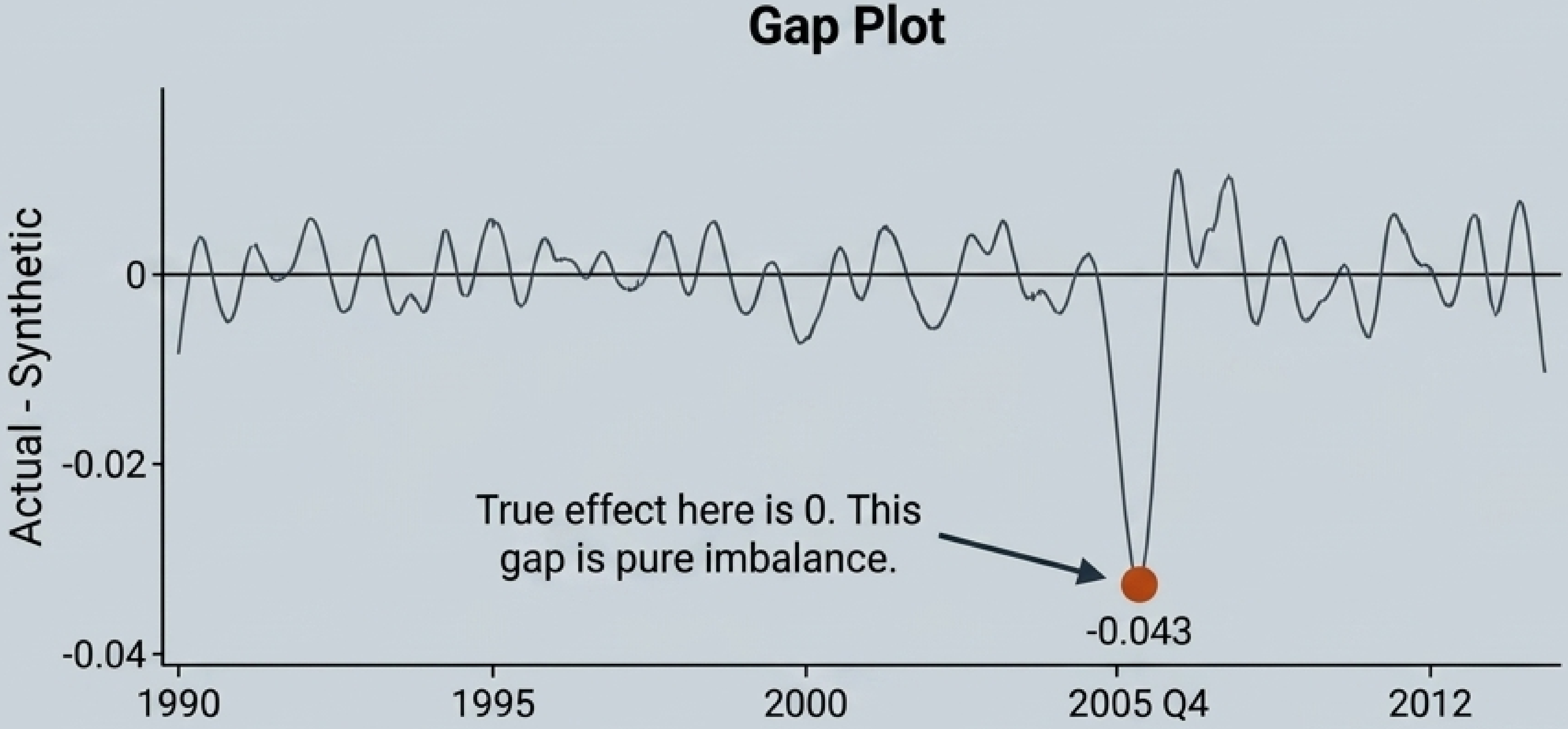
1. Sum of weights equals 1 ($\sum \gamma_i = 1$)
2. Weights must be positive ($\gamma_i \geq 0$)

The Result

Sparse weights: SC: 30%, WA: 22%, TX: 15%, and 42 states get 0%.

If Kansas wanders outside the donor bounds, classic SCM cannot reach it without negative weights.

The Methodological Bottleneck: Pre-Treatment Imbalance



Diagnostic Metrics Panel

Pre-Treatment L2 Imbalance
0.083 79.5% better than uniform weighting, but imperfect
Post-2012 ATT
-0.0294 (≈ -2.9%)

Key Insight

When the pre-treatment fit is imperfect, the post-treatment estimate mixes the true causal effect with matching bias.

The Refinement: Ridge ASCM (Doubly Robust)

Analogy: The Spell-Checker

SCM writes the first draft of the counterfactual. Ridge regression detects systematic errors (imbalance) from the donors and subtracts them.

$$\hat{Y}_{1T}^{\text{aug}}(0) = \sum(\hat{\gamma}_i^{\text{scm}} * Y_{iT}) + \left(\hat{m}_{1T} - \sum(\hat{\gamma}_i^{\text{scm}} * \hat{m}_{iT})\right)$$

The Classic SCM Counterfactual

$$\sum(\hat{\gamma}_i^{\text{scm}} * Y_{iT})$$

The initial baseline drafted by standard synthetic control weights.

+

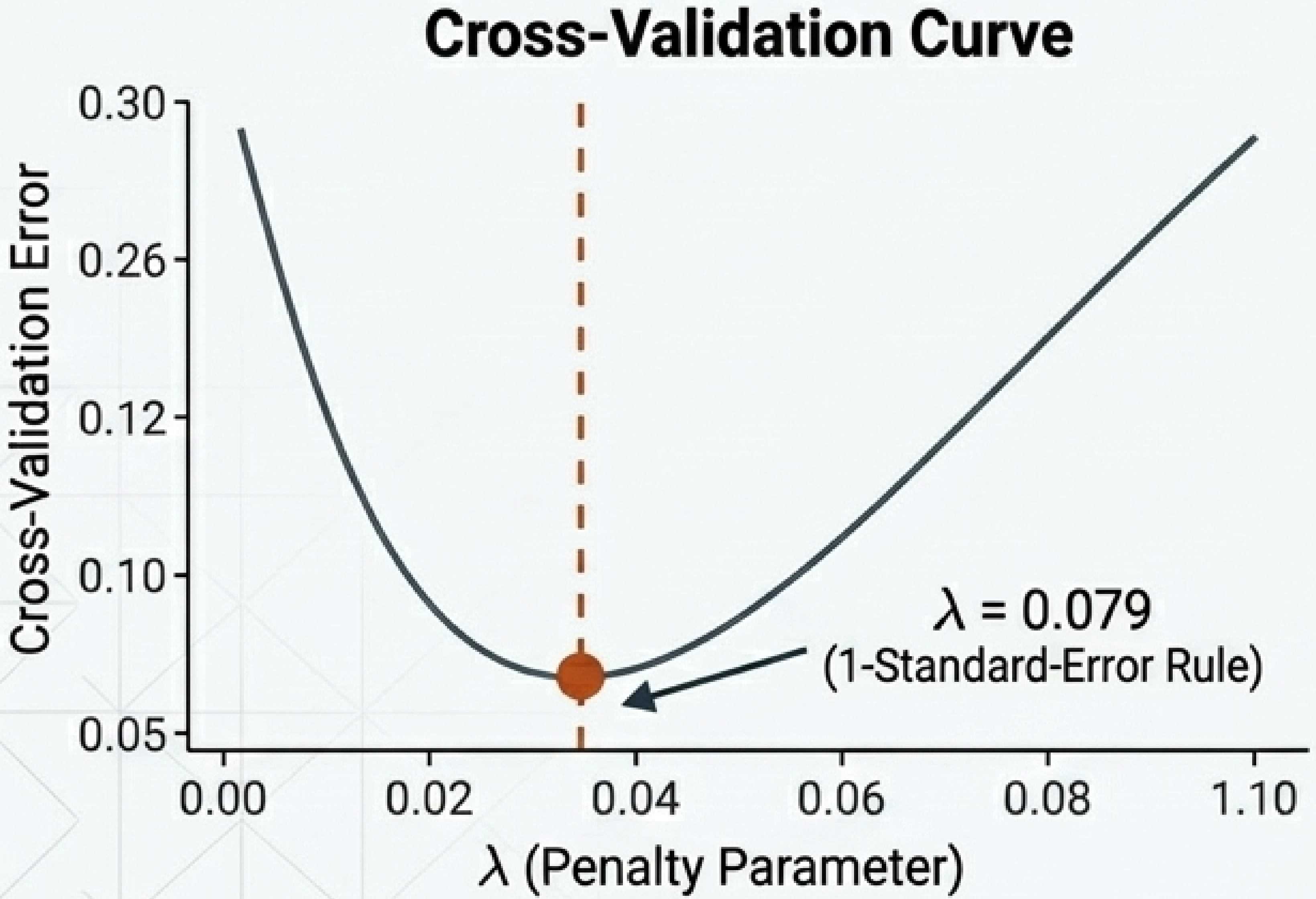
The Ridge Bias Correction

$$(\hat{m}_{1T} - \sum(\hat{\gamma}_i^{\text{scm}} * \hat{m}_{iT}))$$

Outcome model prediction for Kansas minus SCM-weighted outcome model predictions for donors.

Controlled Extrapolation: Bending the Simplex

$$\arg\min_{\gamma} \frac{1}{2\lambda} \|\mathbf{X}_1 - \mathbf{X}_0' \gamma\|_2^2 + \frac{1}{2} \|\gamma - \hat{\gamma}^{\text{scm}}\|_2^2$$



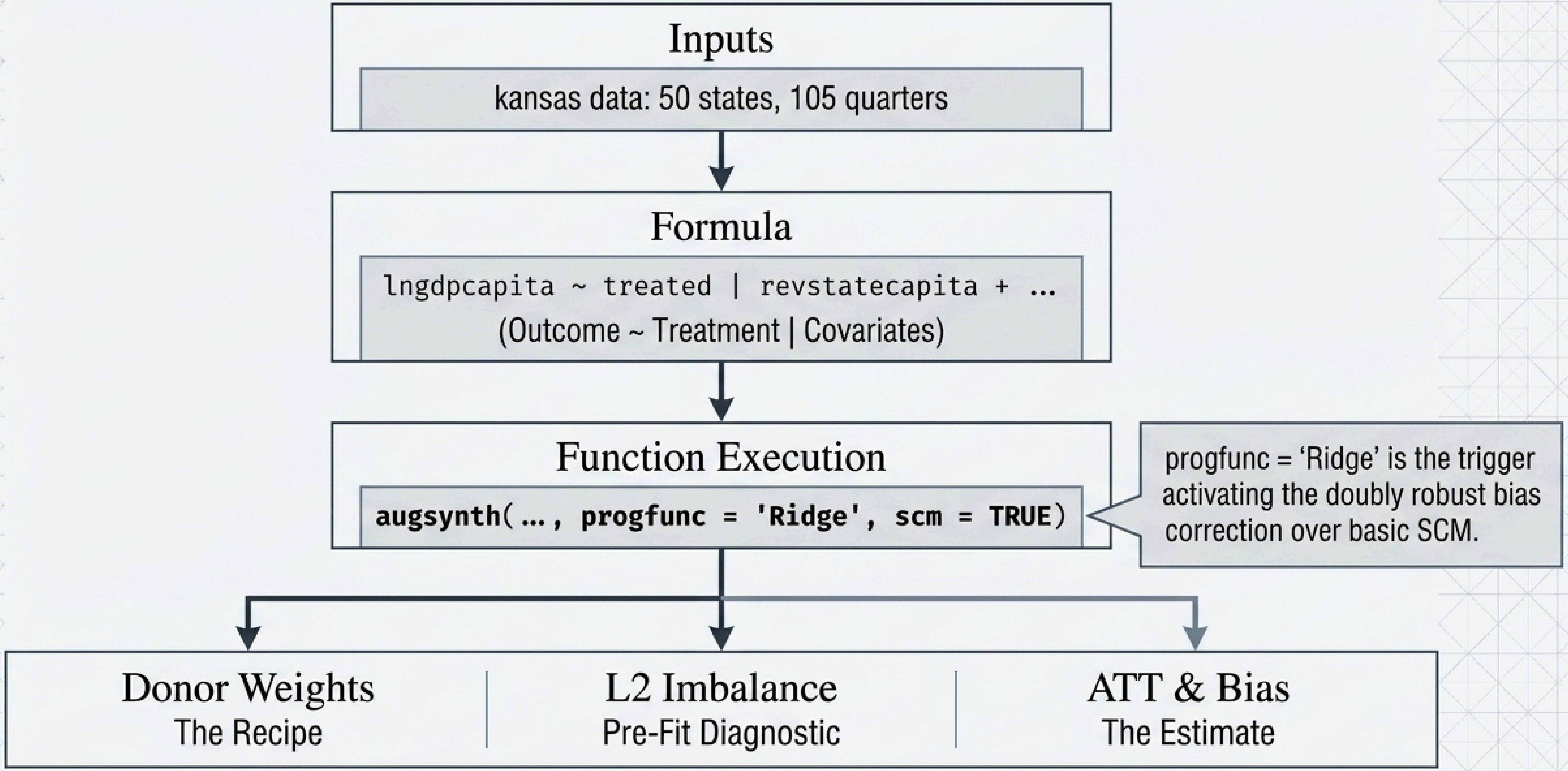
The Dial

- λ controls extrapolation. Large λ stays close to SCM.
- Small λ extrapolates more to improve pre-fit.

The Result

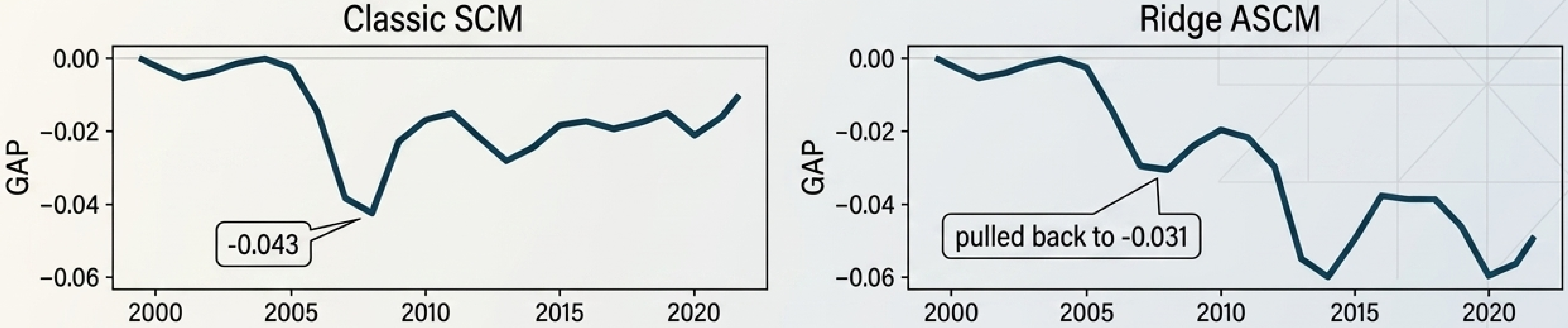
- ASCM assigns small negative weights to 21 donors.
- The root-mean-square change in weights from Classic SCM is only 0.0147.
- We buy a de-biased estimate for a negligible departure from the convex hull.

Implementation Logic: The augsynth() Workflow



Estimator Evolution: Correcting the Mismatch

Gap Plots



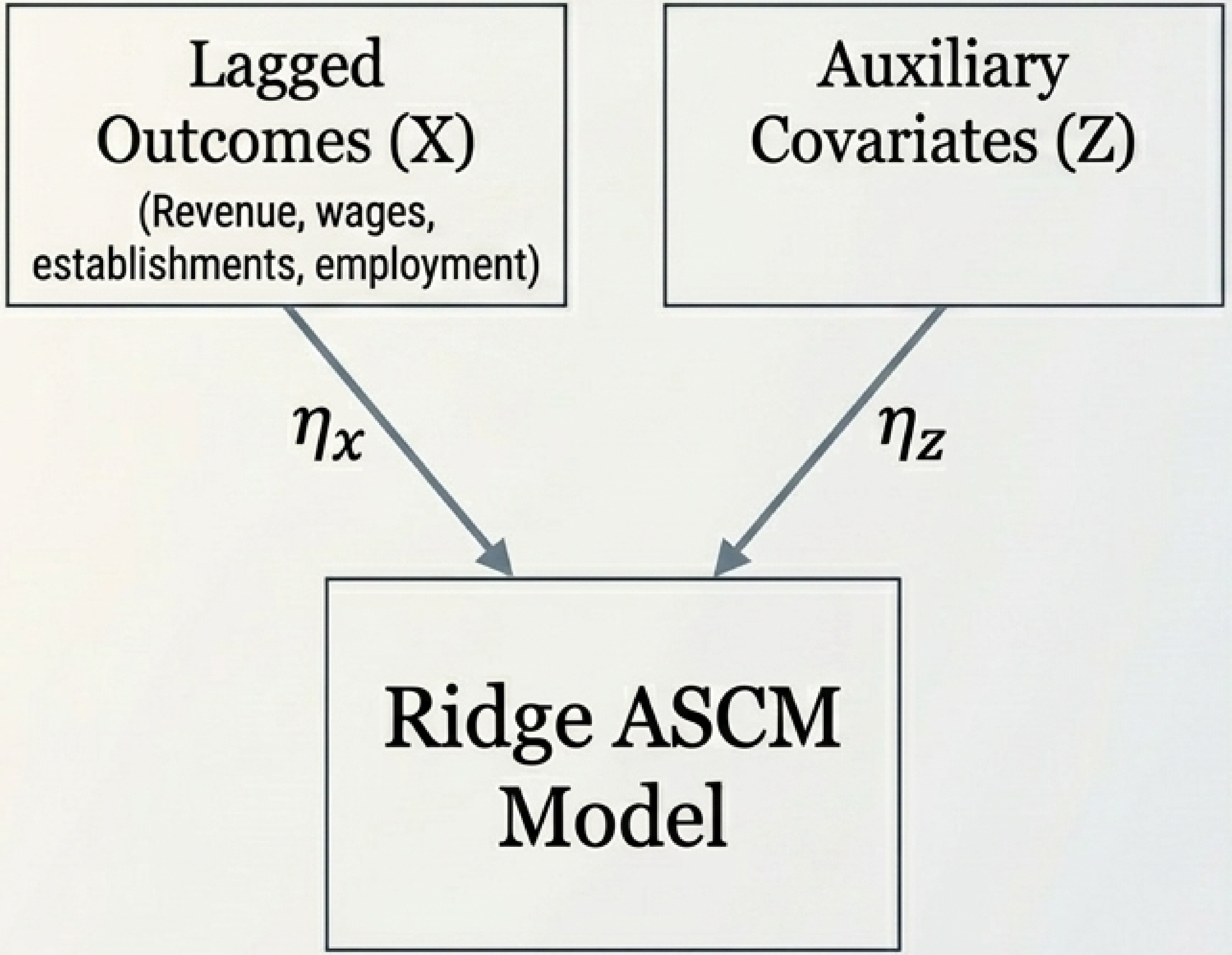
Estimator Evolution Matrix

Metric	Classic SCM	Ridge ASCM
Estimate (ATT)	-0.029	-0.040 (≈ -3.9% shortfall)
Pre-Fit L2	0.083	0.062 (84.7% better than uniform)
Estimated Bias	—	0.011

The model calculates that classic SCM was biased toward zero by roughly one-third of the total effect size.

Expanding the Model: Covariate Augmentation

Dual-Input Funnel



Outcome Imbalance falls to **0.054**
Covariate Imbalance falls to **0.005** (97.7% improvement)
The ATT deepens further to **-0.0609** (≈ -5.9%)

Matching on GDP trajectory alone **misses structural differences. Adding covariates forces the synthetic unit to match the treated unit's underlying economic mechanics.**

Synthesis: The Bias-Correction Ladder

Classic SCM
ATT -0.029 | L2 0.083 | Bias: —

Fixed Effects
ATT -0.034 | L2 0.082 | Bias: —

Ridge ASCM
ATT -0.040 | L2 0.062 | Bias: 0.011

Residualized
ATT -0.055 | L2 0.067 | Bias: 0.006

Covariate ASCM
ATT -0.061 | L2 0.054 | Bias: 0.027

Core Econometric Lesson

The un-augmented SCM estimate was conservative. Progressively de-biasing the estimator monotonically deepens the measured negative effect of the tax cut. Failing to correct pre-treatment mismatch biases the result toward zero.

The Validation Challenge: Is it Just Noise?

A synthetic-control estimate is a difference between two estimated curves derived from exactly one treated unit. Classical standard-error formulas do not apply.

Four ways to ask: Could this gap be chance?

Methodology Groups

Placebo / Permutation Tests

A synthetic-control estimate is adict compreition or counterfactual elements in treated elements.

Conformal Inference

A synthetic-control estimate is a challenges of counterfactual, lin treated elements.

Jackknife+ Over Time

Synthetic-control estimate that must more of counterfactual, for nectrobul over time.

Leave-One-Donor Jackknife

Leave-one-donor jackknife in estimate (from counterfactual, 'or treated elements.

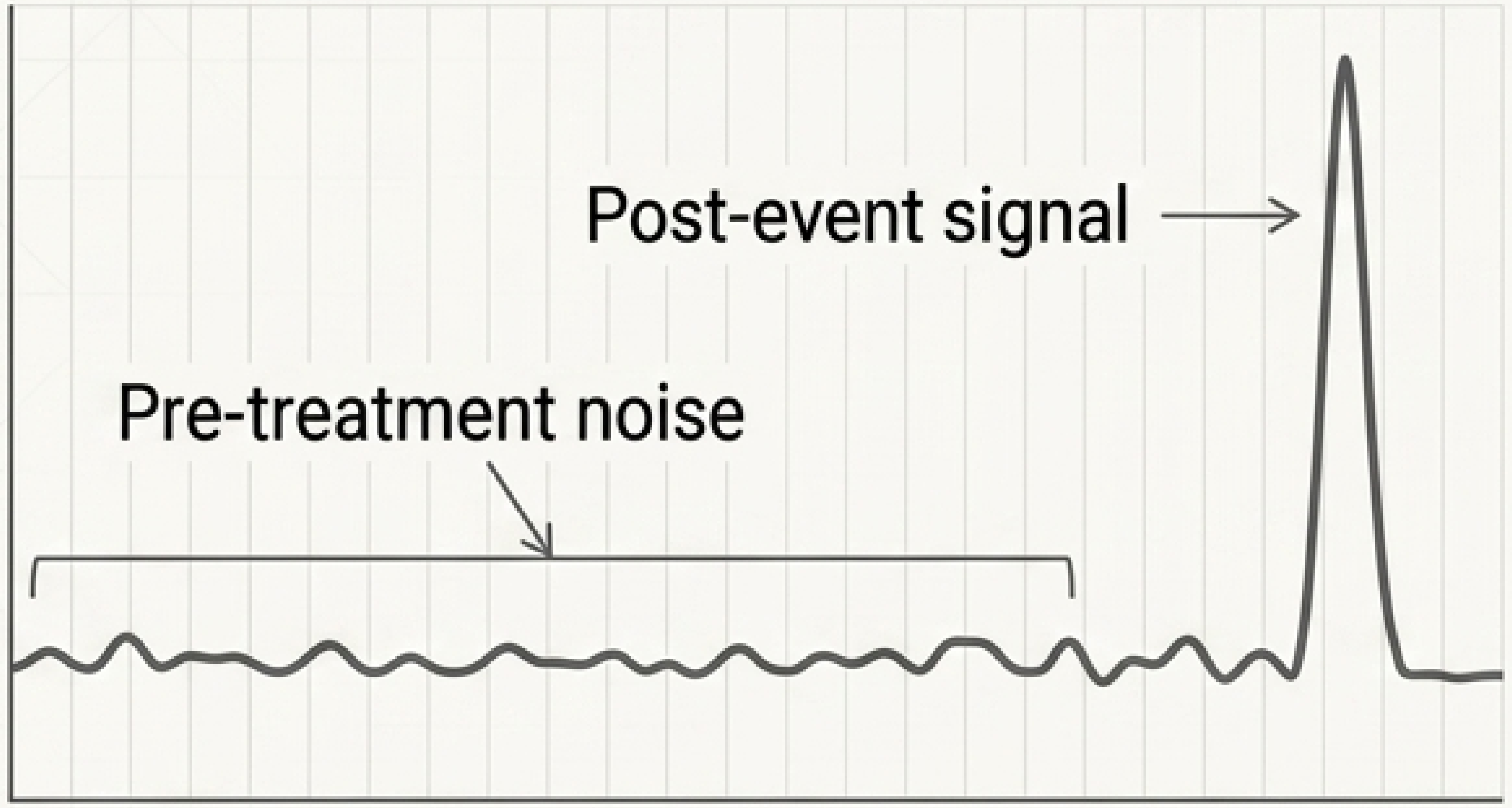
Perturbation Type

Crucial Distinction

These methods will disagree because they probe different sources of variation (perturbing time vs. perturbing units) and make different exchangeability assumptions.

Conformal Inference: The Modern Default

Analogy: The Calibrated Lie Detector



A post-treatment spike only counts as a statistical signal if its magnitude exceeds the normal fluctuation established during the long resting period.

Mathematical Logic & Results

$$p(\tau_0) = \frac{1}{T_0 + 1}\left(1 + \sum_{t=1}^{T_0} 1\{|\hat{u}_t| \geq |\hat{u}_T|\}\right)$$

Tests the sharp null ("effect equals τ_0") by checking if post-treatment residuals $|\hat{u}_T|$ look like typical pre-treatment residuals $|\hat{u}_t|$.

Kansas Result

Joint-null p = 0.066.

Pointwise significance achieved in individual quarters (e.g., 2013 Q3 p=0.024, effect -0.059).

Inference Architecture Matrix: Time vs. Unit Perturbation

Method	Perturbation	Metric	P-value / CI	Kansas Verdict
Permutation	Unit (Fake Treatment)	RMSPE Rank	p = 0.10	Suggestive (Ranks 5th of 50)
Conformal	Time (Pre-period noise)	Residual Distribution	p = 0.066	Borderline (Pointwise significant)
Jackknife+	Time (Leave-one-out)	Spread of errors	CI: [-0.058, -0.021]	Significant (Excludes zero)
Leave-One-Donor	Unit (Drop donor)	Estimate Sensitivity	CI: [-0.088, +0.007]	Not Significant (Includes zero)

Insight
The point estimate (-0.040) remains fixed. The verdict depends entirely on whether we test stability across time or stability across comparison units. Dropping South Carolina (30% weight) shifts the estimate, inflating the LOD error.

The Econometric Verdict: Kansas 2012

The Effect	The Method	The Uncertainty
3% to 6% persistent shortfall in GDP per capita relative to the synthetic baseline, concentrated in 2013–2014.	**Augmenting SCM with Ridge regression and covariates** proves that classic un-augmented models systematically understated the effect due to pre-treatment imbalance.	Statistical significance is borderline (**p ≈ 0.07 – 0.10 for joint nulls**). We report suggestive-to-moderate evidence, not a knock-down result.

The practical takeaway for causal inference is a pattern: the better we engineer the **methodological comparison** to correct imbalance, the larger the measured **negative effect** becomes. A hasty reading of a classic SCM **p-value** would falsely suggest “**no detectable effect**”.