



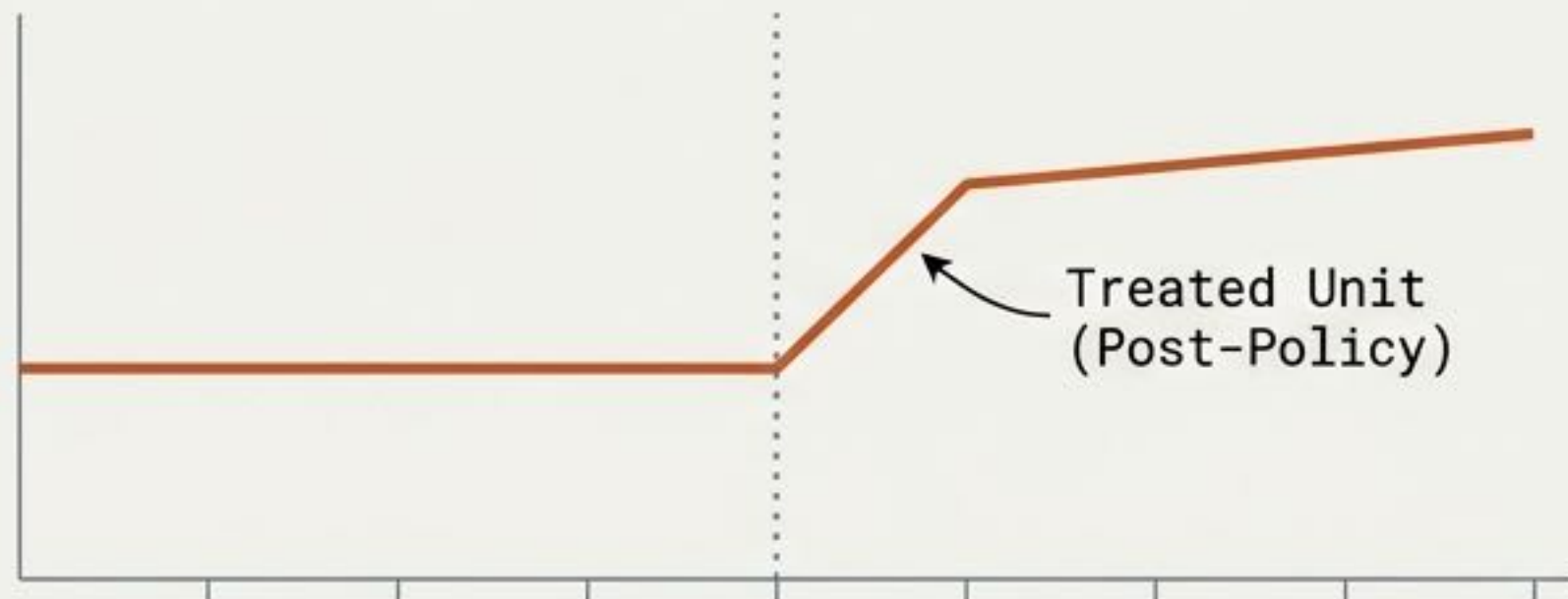
Augmented Synthetic Control for Multiple Countries

A Methodological Study Guide
to the augsynth R Package

Focus: Methodological Foundations, Implementation, and Empirical Replication.

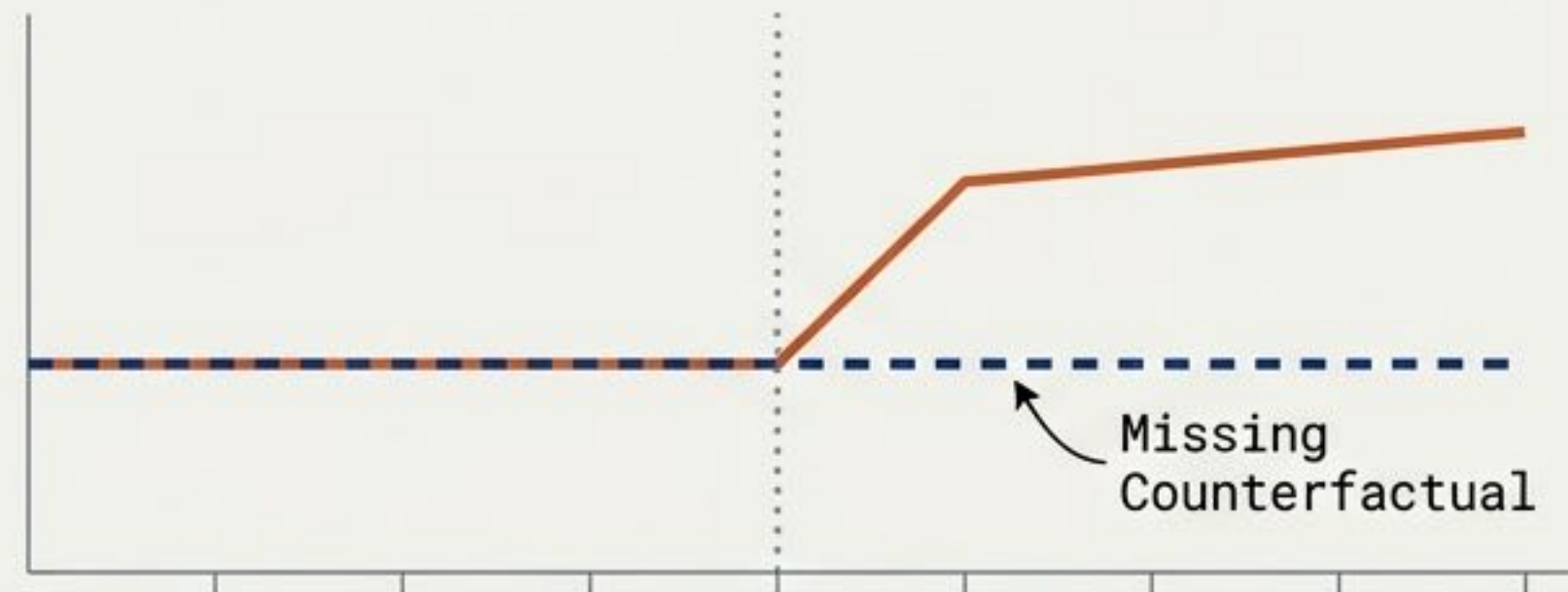
The Fundamental Evaluation Problem

The Observable World



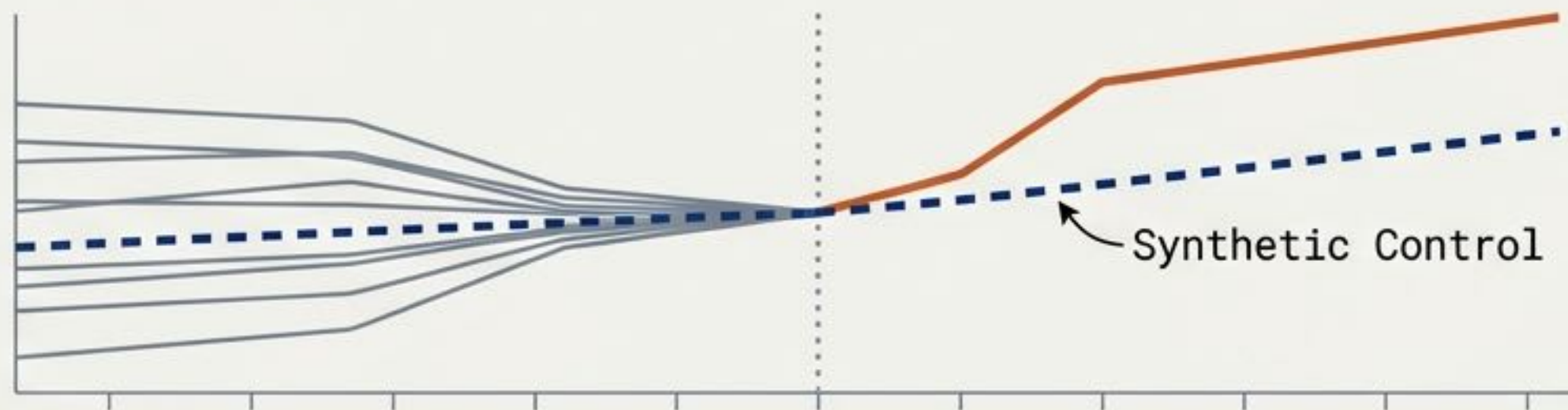
We only observe the world that actually happened. A country adopts a policy, and we track its outcomes.

The Unobservable World



We never observe the path the country would have followed without the policy.

The SCM Solution



Synthetic Control Method (SCM) builds the missing counterfactual from data. It finds a weighted recipe of untreated donor units whose pre-treatment trajectory is indistinguishable from the treated unit's. The post-treatment gap is our estimated treatment effect.

Methodological Lexicon & Intuition

Term	Formal Definition	Conceptual Analogy
Synthetic Control (SCM)	A weighted average of untreated donor units matching the treated unit pre-treatment.	A stunt double assembled from extras. Mimics the star before the dangerous scene (treatment) to show what happens during it.
Augmented SCM (ASCM)	Fits an outcome model on donors and subtracts residual bias when the pre-treatment fit is imperfect.	Tarring a wall, then touching up with a brush. SCM is the broad coat; the outcome model paints over spots the roller missed.
Convex Weights (W)	Non-negative weights summing to 1. Ensures interpolation, not extrapolation.	A recipe summing to 100%. You can blend ingredients but never use a negative amount of flour.
Prognostic Model (progfunc)	Ridge regression on lagged outcomes predicting residual imbalance.	A spell-checker for the counterfactual. SCM writes the draft; Ridge flags and fixes systematic typos.

The Mathematical Engine: Anatomy of ASCM

The SCM Weight Problem

$$W^* = \underset{W}{\operatorname{argmin}} \| X_1 - X_0 W \|_V$$

Finding the donor recipe (W) that best matches the treated unit's pre-treatment vector (X_1).

The Simplex Constraint. Weights must sum to 1 and be ≥ 0 .

The Augmented Estimator

$$\text{Tau}_{aug} = [Y_1 - \sum(w_j Y_j)] - [m(X_1) - \sum(w_j m(X_j))]$$

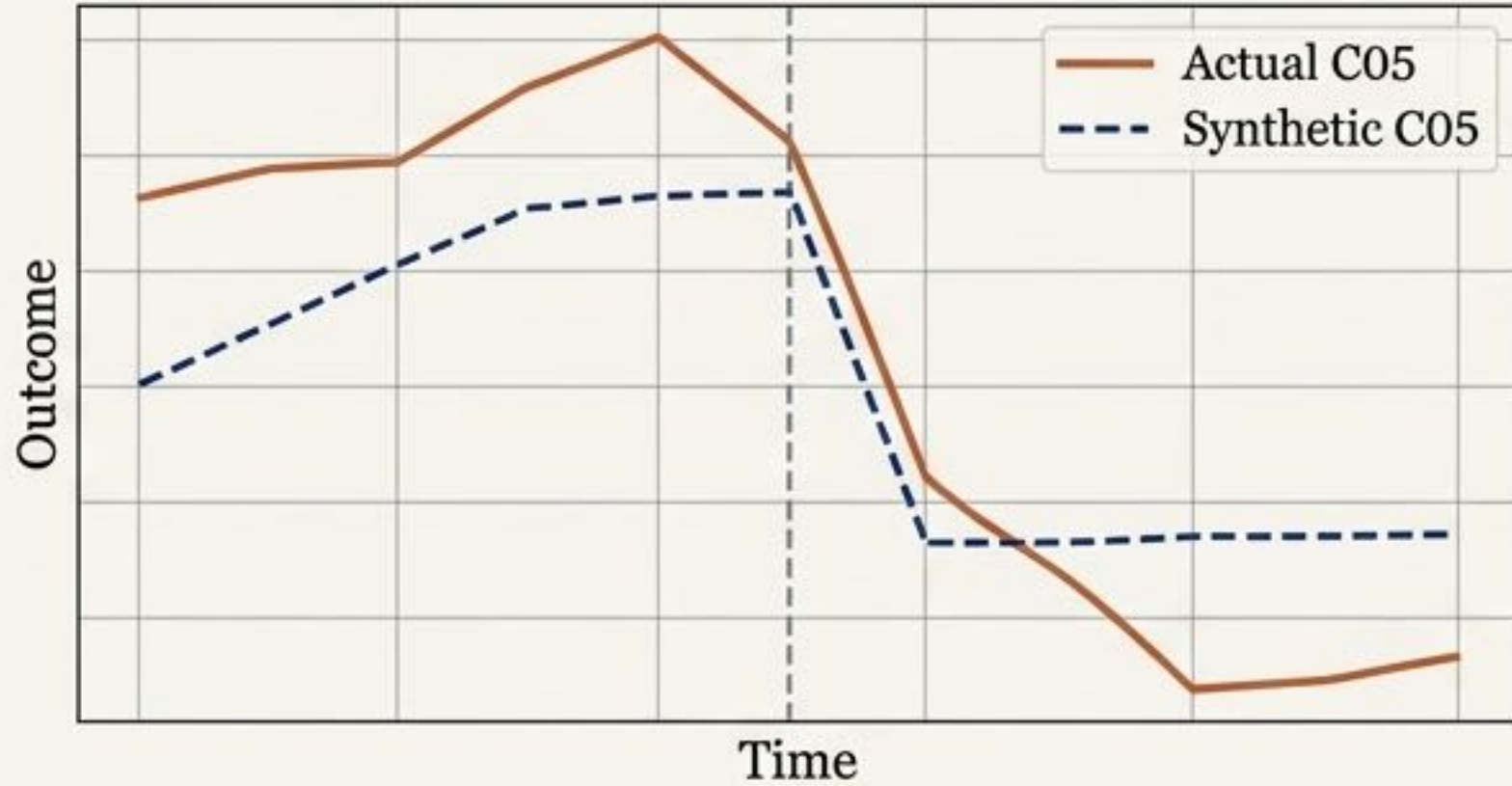
Ordinary SCM Gap. Actual treated minus weighted donors.

Ridge Outcome Model Correction. Removes leftover bias.

If pre-treatment fit is perfect, the donors already reproduce the model, the second term equals zero, and ASCM = plain SCM.

The Diagnostics of Fit: Why Augmentation Matters

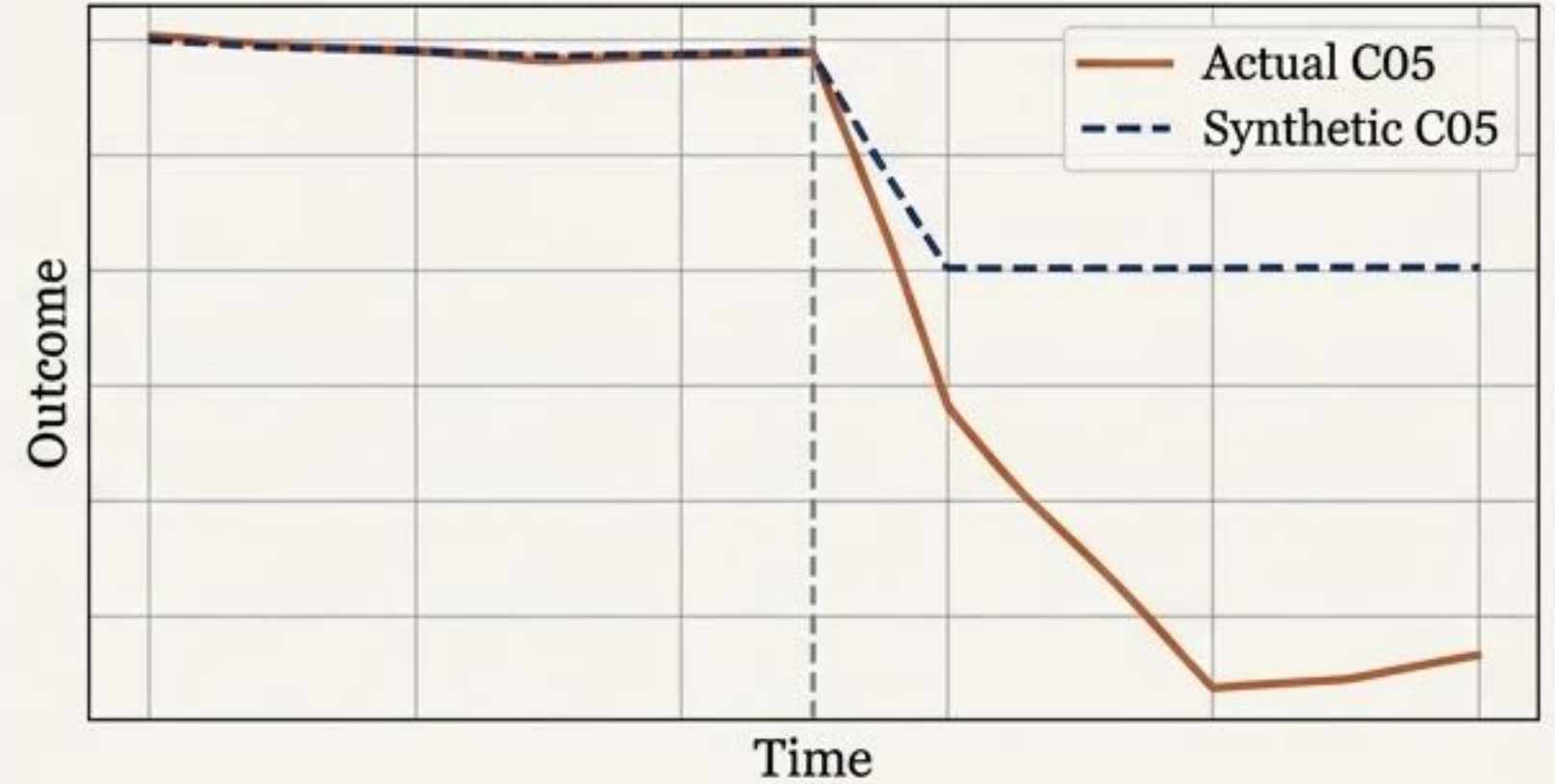
Plain SCM - Poor Fit



The Extrapolation Problem

When a unit sits outside the donor hull, plain SCM cannot match the pre-period. For Unit C05, unmatched bias swamps the signal: true effect is -1.225 , but plain SCM estimates $+0.335$ (the wrong sign).

Ridge-ASCM - Rescued Fit



The Bias Correction

Ridge-ASCM closes the pre-treatment gap, successfully recovering the true negative effect (-1.316).

The Rule of Scaled L2 Imbalance: Always read the pre-treatment fit. If Scaled L2 is large, plain SCM is failing. In this simulation, augmentation drops mean recovery error across all units from 0.390 to 0.127 .

The augsynth Package Toolkit

`outcome ~ treatment | covariates`

Unit and time passed as arguments; treatment is a 0/1 indicator.

Function	Use Case	Inference
<code>single_augsynth()</code>	1 Treated Unit, Single Intervention Time (<code>t_int</code>).	Conformal Inference (<code>inf_type = 'conformal'</code>).
<code>multisynth()</code>	Many Treated Units, Staggered Adoption (Infers adoption automatically, no <code>t_int</code>).	Wild Bootstrap (<code>inf_type = 'bootstrap'</code>).
<code>augsynth_multiout()</code>	1 Treated Unit, Multiple Joint Outcomes (<code>y1 + y2 ~ trt</code>).	Conformal Inference (<code>inf_type = 'conformal'</code>).

Implementation Workflow

1

Format Panel

Ensure data includes unit identifiers, time identifiers, and a precise 0/1 binary treatment indicator column that turns on when treatment starts.

2

Check Suitability

Run plain SCM (progfunc = 'None').
Read the Scaled L2 Imbalance. If it is high, your unit sits outside the donor hull.

3

Apply Augmentation

Swap to progfunc = 'ridge'.
The outcome model will estimate residual imbalance and apply the doubly-robust bias correction.

4

Execute Inference

Run `summary(fit, inf_type = '...')`.
Ensure you match conformal inference to single-unit functions and wild bootstrap to multi-unit functions.

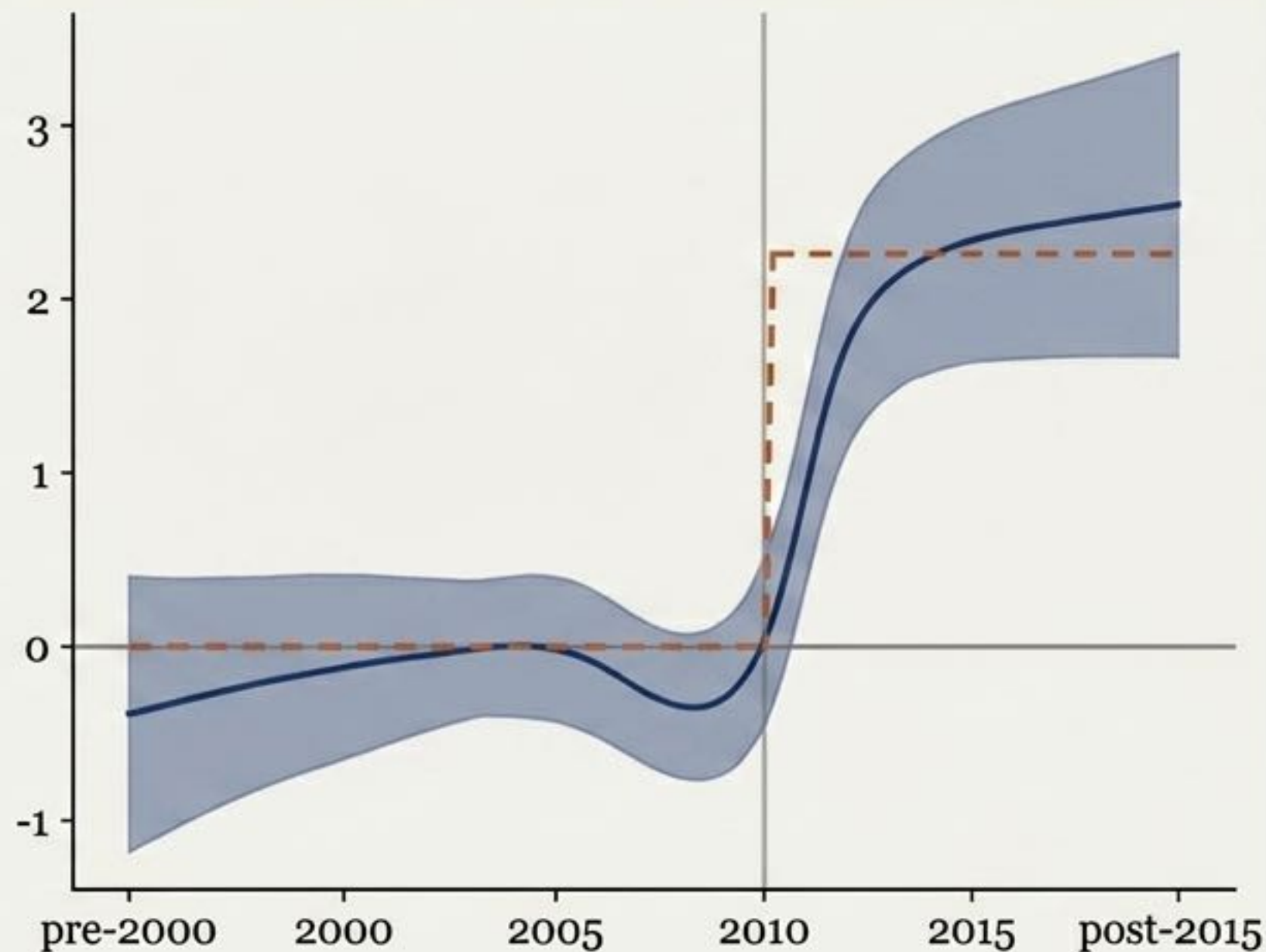
Scenario A: The Single Unit Baseline

```
augsynth(gdp_index ~ trt, unit, time,  
data=c01_data, progfunc='ridge')
```

Recovered Effect: **+2.651**
(True: +2.600, **2% error**)

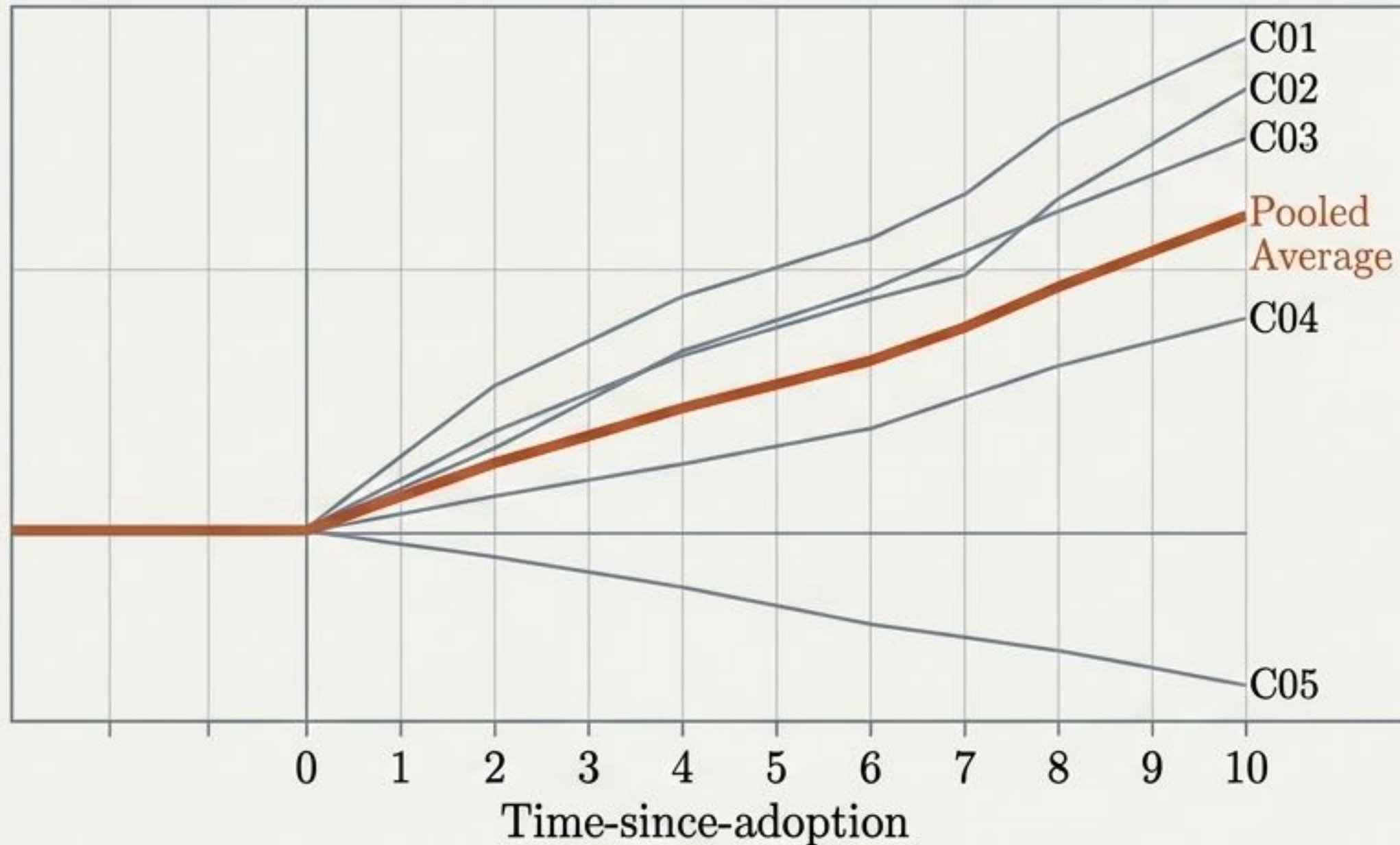
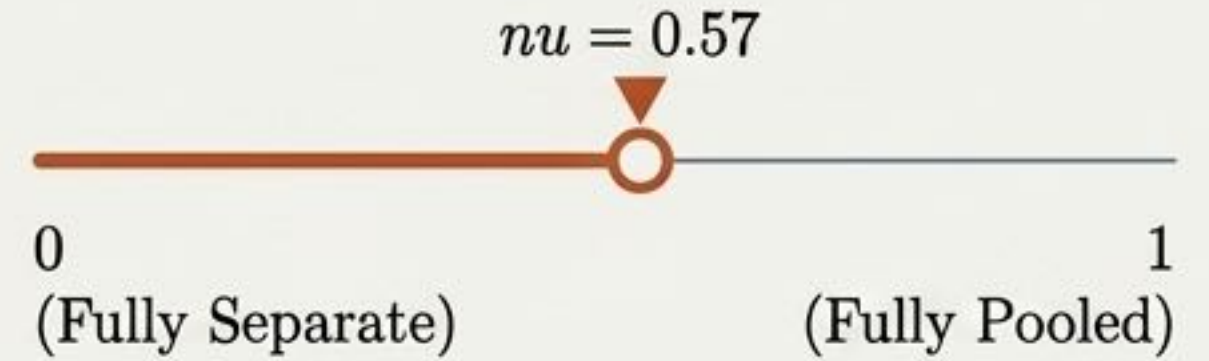
Donor Weights Recipe

C14	23%
Co8	21%
C10	21%
Co6	19%
C16	15%



Sparse weights make the counterfactual auditable; the pre-period gap hovering at zero is the visual signature of trust.

Scenario B: Staggered Adoption & Partial Pooling



Analogy: Grading with a rubric.
Partial pooling borrows strength from the global average to stabilize per-unit estimates.

Pooled Average Effect:
+0.718 (True: +0.735)

C05 Trajectory: **-1.468**
(True: -1.225)

The algorithm recovers both positive trajectories and C05's negative trajectory simultaneously.

Scenario C: Joint Multiple Outcomes

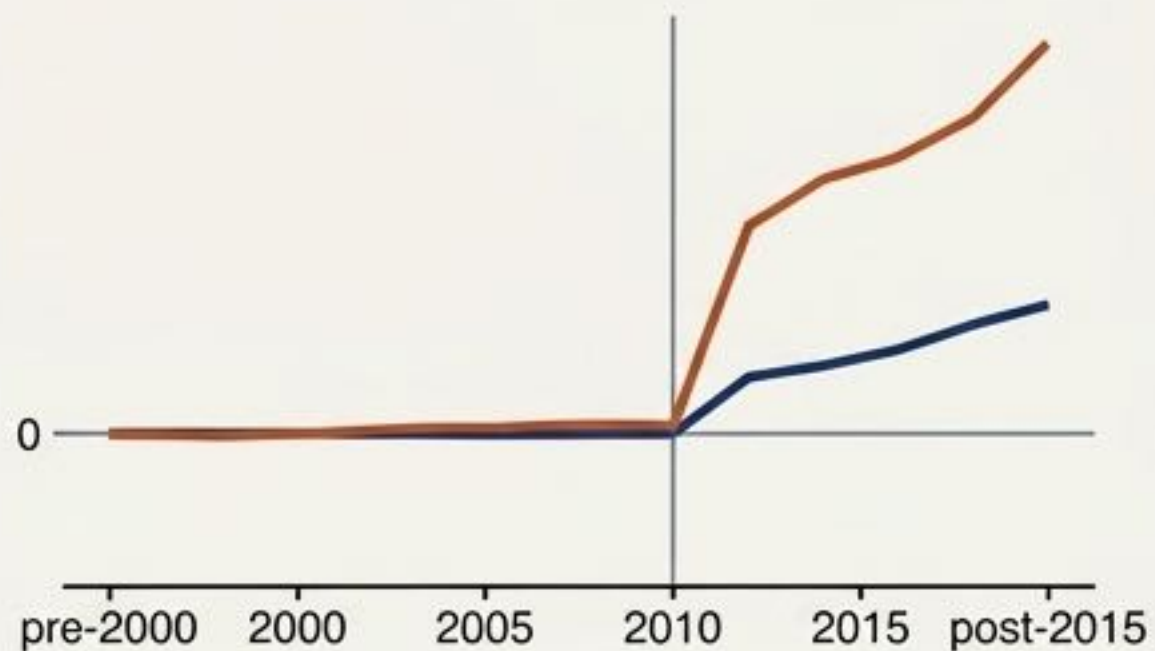
The Syntax & Theory

```
augsynth_multiout(gdp_index  
+ trade_index ~ trt, ...)
```

A single set of donor weights balances all outcomes jointly, borrowing strength across correlated series.

Analogy: One tailored suit fitted to chest, sleeve, and waist measurements simultaneously.

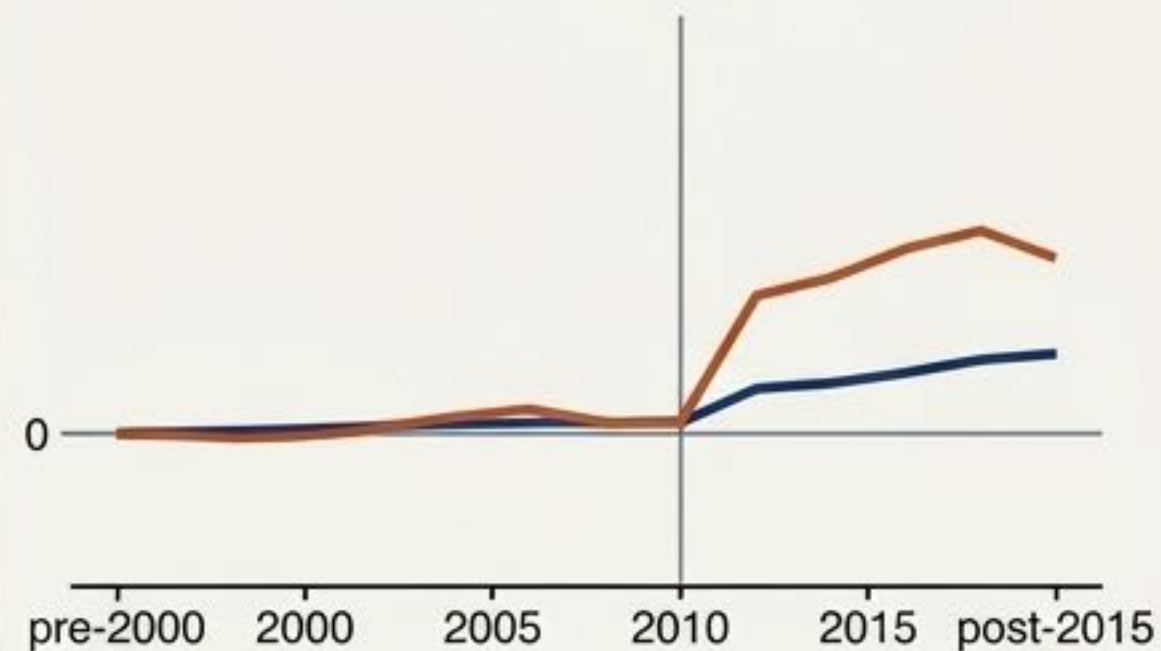
Outcome 1 (GDP)



Recovered effect: **+2.709**

True effect: +2.600

Outcome 2 (Trade)



Recovered effect: **+1.720**

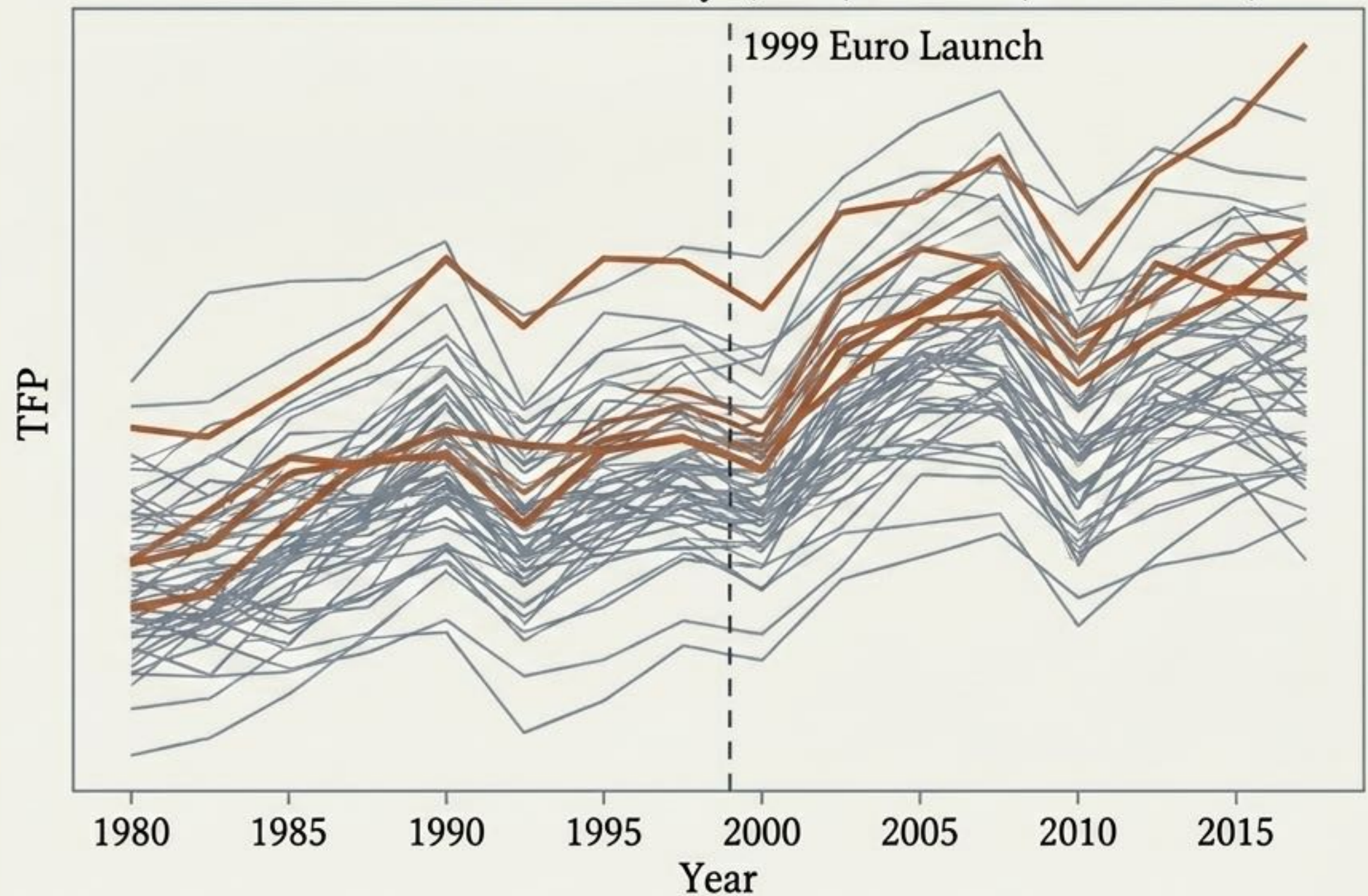
True effect: +1.560

Correlated at 0.6x of the GDP effect. Both recovered within 0.16 of ground truth using a single, unified donor recipe.

Real-World Case Study: EMU & Productivity

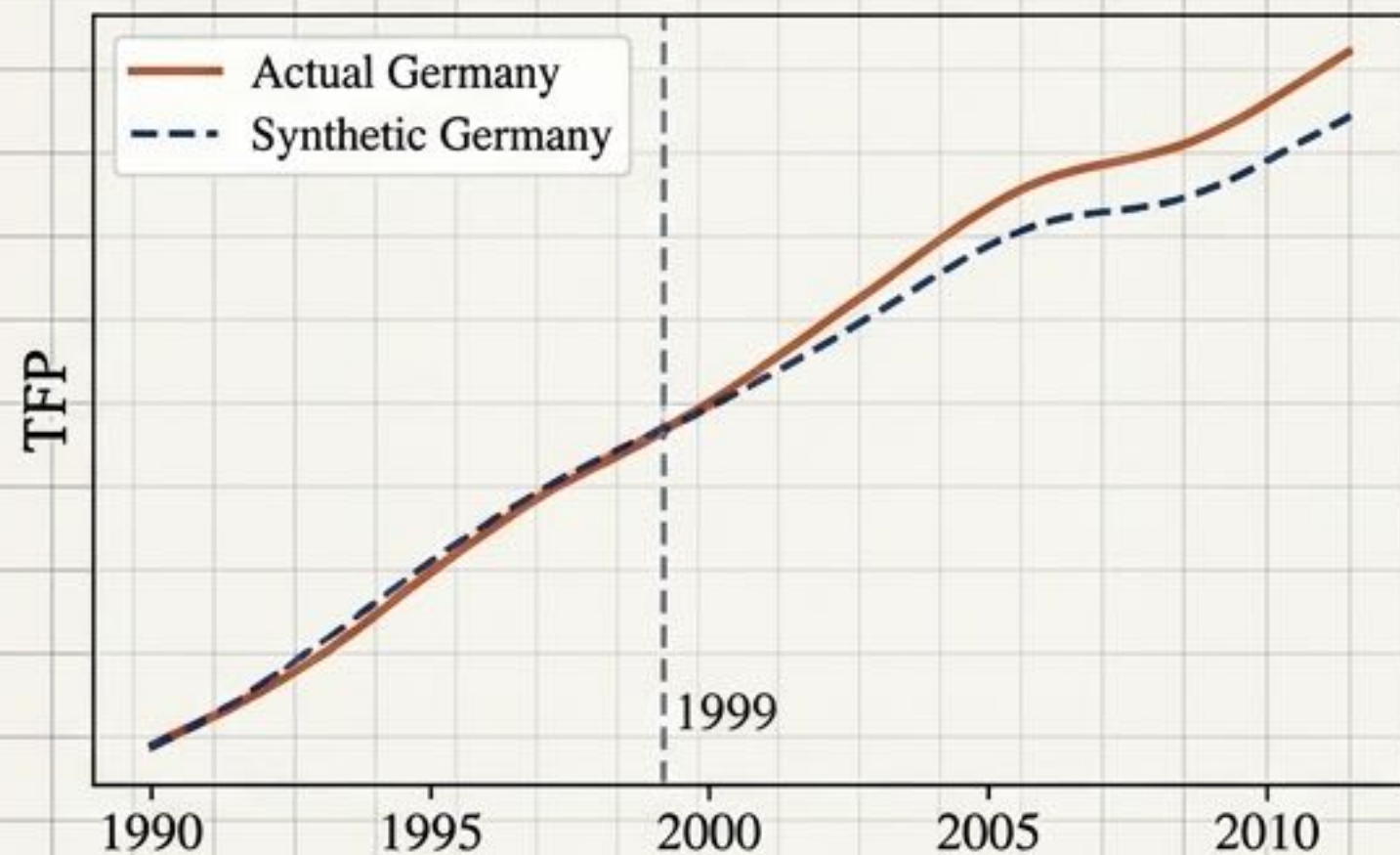
- Research Question: Did joining the euro make founding member countries more productive?
- Data Source: Papaioannou (2021) Replication. 36 countries, 1980-2017.
- Treatment: 12 Euro Founders (intervention in 1999). 24 Non-Euro Donors.
- Outcome: Total Factor Productivity (TFP) and log Productivity Gap vs. USA.

Total Factor Productivity (TFP) Paths (1980-2017)



Empirical Application: Synthetic Germany

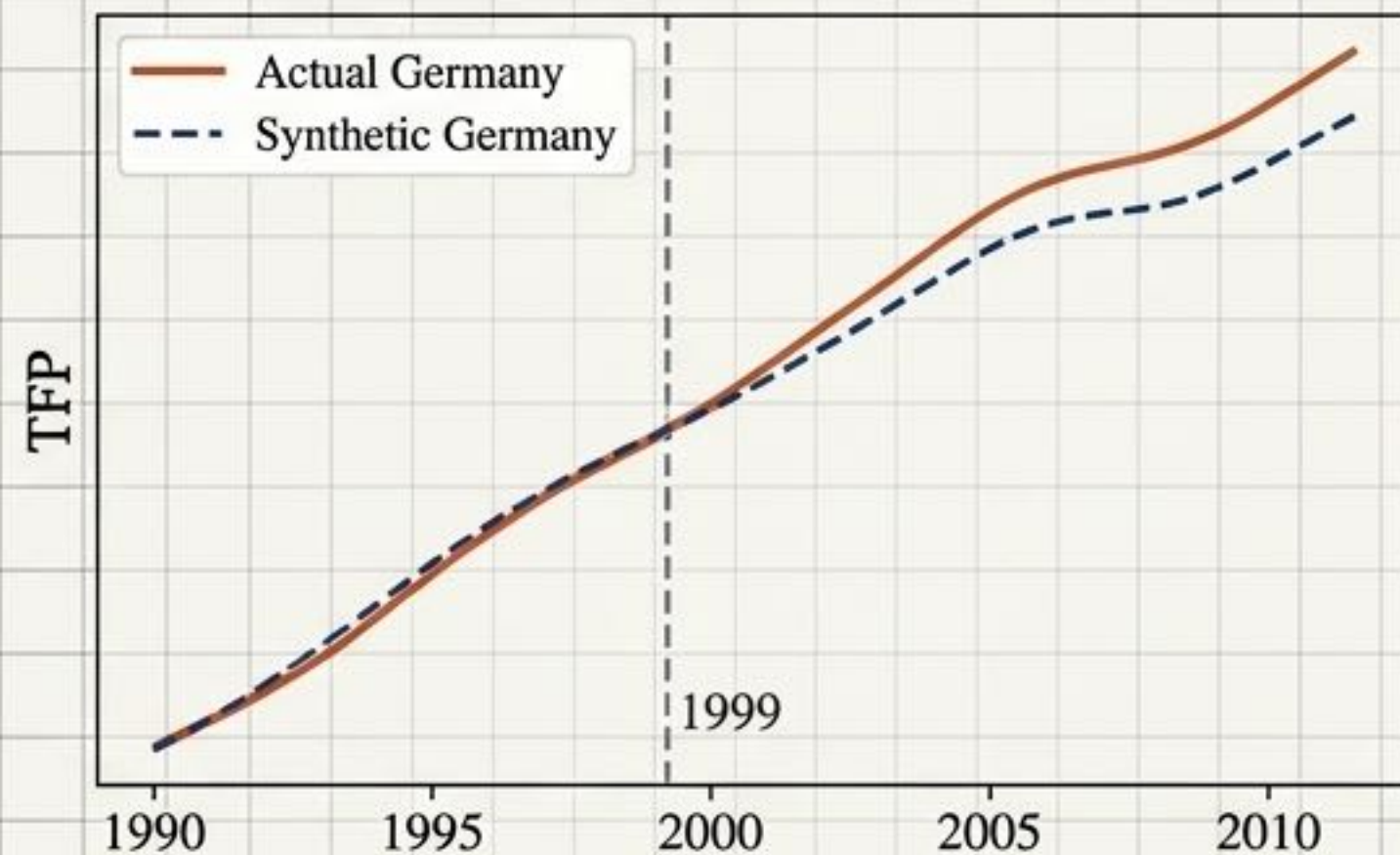
Plain SCM



Average TFP effect: +0.133

Pre-treatment fit is inherently excellent (Scaled L2 = 0.30).

Ridge-ASCM



Ridge-augmented estimate: +0.127

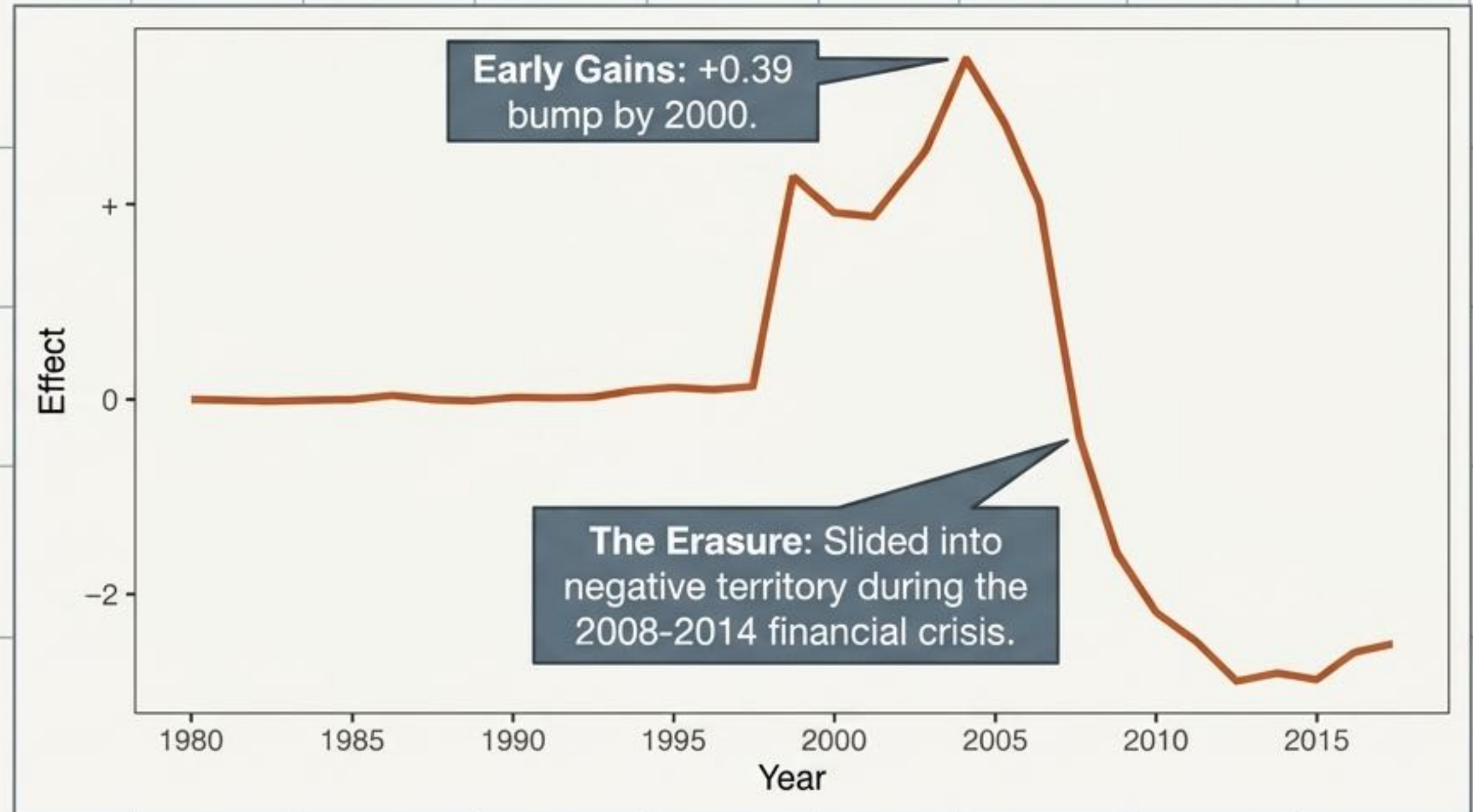
Scaled L2 barely moves (0.301 -> 0.292).

The Insurance Principle: The Ridge-ASCM estimate is nearly identical to plain SCM. Augmentation is an insurance policy. A quiet premium here means the classic Abadie estimate was already trustworthy for Germany.

Interpreting Heterogeneity & Dynamics

WARNING - Flawed Statistic

Pooled Average Effect: -0.016
Misleading if taken at face value.



Rule for Students: Averages hide dynamics in multi-country work. The early gains and crisis losses mathematically canceled out. Always plot the dynamic path.

Robustness Checks in Practice

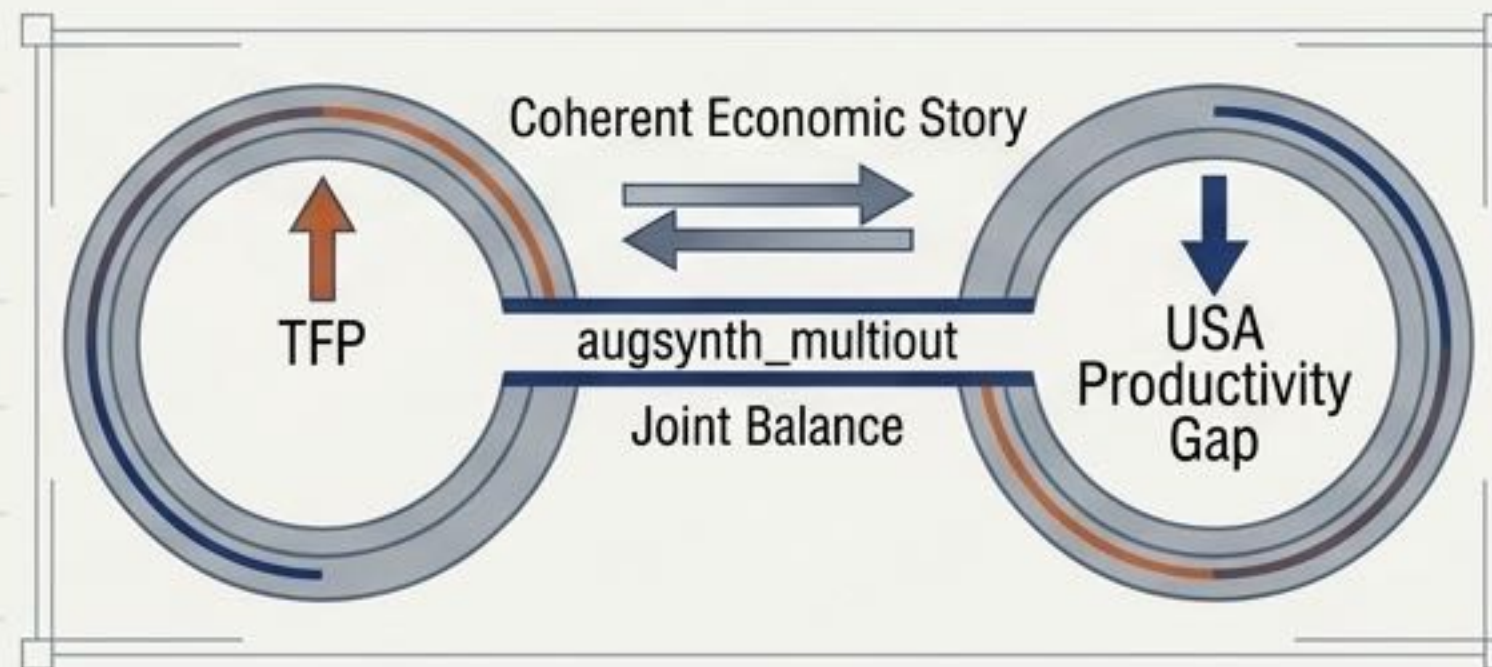
The Anticipation Check (Temporal)

- **Concept:** Markets may have anticipated the euro at the 1992 Maastricht Treaty, not the 1999 launch.
- **Test:** Rerun Synthetic Germany with `trt92` indicator.
- **Result:** The causal reading holds. The estimate is robust (+0.138 vs. the original +0.133).



Joint Outcomes Check (Methodological)

- **Concept:** Does the policy logic hold across related metrics?
- **Test:** Balance TFP and the USA Productivity Gap simultaneously using `augsynth_multiout`.
- **Result:** A single tailored synthetic Germany shows TFP rises (+0.116) AND the productivity gap to the US narrows (-0.151). The economic story is coherent.



Synthesis & Best Practices for the Applied Researcher

1	Validate on Truth	A causal estimator you cannot reproduce on simulated ground truth is a leap of faith. Always test estimators on simulated known-effect data before deploying them on real-world datasets.
2	Read the Imbalance	Augmentation rescues bad pre-treatment fits but does no harm to good ones. Look at the Scaled L2 Imbalance every time. If it is small, classic SCM is fine; if it is large, trust the Ridge correction .
3	Report Honestly	Do not obscure heterogeneity. In multi-country studies, an average treatment effect can hide profound dynamics (like the EMU's +0.39 boom and subsequent crisis crash). Show the dynamic path and per-unit spread.